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## Shift scheduling with linear programming: A case study in a fire station

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**Abstract.** Due to demographic variations and climate changes, the risks of natural disasters are increasing. Therefore, with the risk of fire and other public welfare risks, firefighters are necessary to protect people in a community and its assets, responding to emergencies, quickly and effectively. This way, it is necessary to have a sufficient number of firefighters in a fire station, to give a reliable answer to any emergency that could occur. However, human and economic resources are

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scarce. In this paper, a linear programming approach was developed in order to determine the best shifts of the working week, minimizing the number of firefighters necessary to satisfy the already registered and emergency tasks. The results show that, with the solution obtained, there is a slack between the solution found and the reality applied in the fire station, which could be improved by changing their shifts and reducing the number of firefighters for each shift.

**Keywords:** Schedule planning; Fire station; linear programming; shifts.

## 1. Introduction

The civil protection activity has a permanent and multidisciplinary nature, and it is up to all public administration bodies to promote the indispensable conditions for its execution. It includes tasks such as rescuing people and other living beings in danger, protecting cultural and environmental assets and values, and supporting the restoration of normality in people's lives in areas affected by serious accidents or catastrophes [1].

Currently, we can consider that Volunteer Firefighters, as civil protection agents, form the basis of the organization of relief to the populations in Portugal. These humanitarian associations of firefighters, which are found throughout the country, provide a crucial service to society, especially due to their proximity to local populations. In this way, they allow and guarantee that this type of service is provided to local population with great effectiveness, quality, and efficiency. However, the support that firefighters provide still has some shortcomings, as there is an increased difficulty in distributing qualified professionals in the necessary shifts.

In this work, an initial approach is made to the shifts' scheduling, on weekdays, in a fire station in the North of Portugal, considering emergencies and pre-established work agreed with entities outside the fire department.

Thus, in Section 2, a literature review is made regarding the importance of volunteer firefighters and simultaneously shift work. In Section 3, the methodology of this work is presented, presenting the case study of the Volunteer Firefighters of Paredes, as well as the linear programming for the formulation of the problem. In Section 4, the mathematical problem is presented, and in Section 5 the results obtained are shown. Finally, in Section 6, the main conclusions are drawn, as well as the potentialities of the problem for future work.

## **2. Literature review**

### **2.1. Volunteer firefighters**

A firefighter is any individual who, “professionally or voluntarily integrated into a fire department, has the task of fulfilling the missions of the fire department, namely the protection of human lives and property in danger” [2]. The importance given to quick and effective assistance is increasing, as the risks of natural disasters are increasingly common. There is a new reality caused by demographic variations and major climate changes, so in addition to relief, there is also a vision for prevention and risk analysis [2].

The firefighter profession is a high-risk activity, and it is necessary to overcome several levels and acquire a set of knowledge, not only technical, but on how to act and be in society, in order to be able to act quickly and effectively. Over the years, the mission of firefighters has been transformed, due to changes in the way of life of the population and due to technological advances. Currently, this mission is based on the relief of various situations [3], namely:

- natural hazards;
- technological risks, such as urban fires, transport fires; road accidents and industrial accidents (gas leaks);
- mixed risks, such as rural and forest fires, debris fires and total or partial safety compromise (falling trees; flooding);
- protection and assistance for assets and people, such as health care, intervention in legal conflicts and assistance and prevention of human activities;
- operations and alert states verified when official trips or training and provision of resources are necessary;
- non-urgent transport, including the transport of patients considered non-urgent.

### **2.2. Shift work**

Shift work, as we know it today, dates to the end of the 18th century, however it has undergone changes and modifications over the years [4]. Increasingly, the shift schedule is used in several service providers, due to the decrease in the number of hours of work per week legally allowed globalization and technological advances. Shift work can be defined as “a way of organizing daily working hours in which different persons or teams work in succession to cover more than the usual 8 hours per day, up to and including the whole 24 hours” [5].

To guarantee the continuity of the provision of services, the succession of several work groups in their work functions is necessary. Groups can have rotating schedules or fixed schedules. Currently, some companies work continuously for 24 hours, maintaining uninterrupted production or provision of services, while others paralyze their activity for some period [6]. It is also relevant to explain the various forms of shift work and their differences. There is fixed shift work, in which the worker always works the same schedule, and there is rotating shift work, and the work schedule is changed between the various employees [4]. Under a fixed shift system, working time can be organized into two or three shifts: the early, late, or night shifts. Under a rotating shift system, workers might be assigned to work shifts that vary regularly over time; these are called “rotating shifts” because they rotate around the clock (e.g. from a shift in the morning to one in the afternoon, to one at night) [7].

The discussion around this topic is constant since it can contribute to countless consequences for the worker. The worker’s adaptation to work and work shifts, especially when there are constant changes, can be quite complex, repeatedly leaving him under an environment of stress and frustration. It may cause fatigue and drowsiness, due to the difficulty of adjusting to them. In addition to all these aspects, this type of work could have a significant impact on the social life of workers ([8]; [9], [10]). All the effects mentioned above may negatively influence employees in the performance of their duties, reducing efficiency and productivity, and may also be more susceptible to making mistakes [11]. The impact of shifts on firefighters is also a study object. Shift work can be associated with both short and long-term cognitive impairment [12], associated with sleep problems. Family support can make up for the lost resources of demanding shifts, and the lack of time for the family could contribute to conflicts and emotional exhaustion [13].

### **3. Methodology**

The case study will be based on the Volunteer Firefighters of Paredes, in the North of Portugal, and the methodology developed to solve the problem of scheduling work is based on a mathematical model of linear programming. In this way, it is intended to ensure the optimization of shifts at the fire station without jeopardizing the efficiency of each of the shifts.

#### **3.1. Paredes Volunteer Fire Station**

The Volunteer Firefighters of Paredes were created in June 1884 and today have the status of Institution of Public Utility. Since 2009, a protocol for the Medical Emergency Station installation has been signed, composed of two rescue ambulances

and two multiple transport ambulances, to serve the citizens of Paredes [3]. At the Paredes Fire Station, firefighters can be split into two categories: professionals and volunteers. Volunteer firefighters only perform their function after work hours and are not remunerated, being less often at work. Professional firefighters, 16 in total, are paid for the service provided and have a fixed schedule to work at the firehouse. Professional firefighters operate from Monday to Friday, performing duties between 07:00h and 21:00h. Only the call centre team has different hours, given the need for this service to be available 24 hours per day and seven days a week. Within the category of professionals, there are four distinct teams: call centre operators; transport of non-urgent patients; rescue ambulance team, and permanent intervention team. The distribution of personnel by shifts will be described in the next section.

### 3.2. Linear Programming

There are several methods used to determine the better solution for schedule problems namely, Markov decision process and dynamic programming, large neighborhood search, genetic algorithms, linear programming, simulation, heuristics and metaheuristics [14].

Linear programming is a tool recurrently applied in several studies, which uses a mathematical model to describe the problem being studied. The model forces all mathematical functions used to be linear. On the other hand, the word “programming” refers to the activity planning to achieve the best possible result. Hillier and Lieberman [15] considers that this model is very useful for the allocation of resources and activities when they are scarce and limited. The author characterizes it as a mathematical tool, in which through a linear function, it is possible to reach the objective, be it maximize or minimize. Mathematically, a linear programming model is based on an objective function, that is, the goal to be achieved, and on existing restrictions on the objective function. Considering the decision variables (non-negative, the solution of a linear programming problem boils down to finding the optimal value (maximum or minimum, depending on the problem) of the linear expression [16]:

$$F = cx_1 + c_2x_2 + \dots + c_nx_n \quad (1)$$

subject to restrictions expressed in inequalities:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \geq b_1 \quad (2)$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \geq b_m \quad (3)$$

The constants , e they are determined by capabilities, needs, costs, profits, and other requirements defined by the constraints of the problem. The problem was solved using Microsoft Excel software, using the Solver package. The solver is one of the programs called hypothetical analysis tools, since it allows finding an optimized solution for a given problem or equation [17]. It is a non-specialized, widespread and easy to access software program, and could work until 200 variables, which could be a good opportunity to tackle some simple optimization problems.

#### 4. Application of the study: problem formulation

To solve this problem an initial approach was taken to the shifts' schedules. The following assumptions were made:

- only professional firefighters were considered, disregarding the voluntary work because it is already defined taking into account the restrictions of each volunteer firefighter;
- only working week shifts was study into account for the purposes of programming, which could lead to some incomplete shifts as they would have to include hours intended for weekend shifts;
- any professional firefighter has the competence to develop any of the four tasks previously defined, not considering any specialization or preference in specific tasks.

Considering the information collected at the fire station, the day was divided into blocks of 3 hours during the day and a block of eight hours during the night:

$[7h - 10h]$ ,  $[10h - 13h]$ ,  $[16h - 19h]$ ,  $[19h - 21h]$ , and  $[21h - 07h]$ .

Additionally, and according to the history of working hours defined by the fire station, five possible shifts are foreseen (variables under study):

$t_i$ : number of professional firefighters working on shift  $i$  ( $i = 1, 2, 3, 4, 5$ )

where  $t_1 = [7h - 13h]$ ,  $t_2 = [7h - 16h]$ ,  $t_3 = [10h - 19h]$ ,  $t_4 = [13h - 21h]$ , and  $t_5 = [20h - 7h]$ .

It should be noted that each day, all shifts, except , includes a one-hour break for meals, ensuring continuity of service permanently. In this naïve approach to the shift problem, and for comparisons reasons, the authors opted for letting the same shift as defined previously for the fire station.

Considering the minimum number of professional firefighters required in each hourly block, based on previously agreed services and the minimum number of firefighters needed to respond to an emergency, the following restrictions were defined:

- the total number of firefighters must be less than or equal to 16, taking into account the existing human resources in the firehouse;
- the minimum number of firefighters in the hour block [7h-10h] should be 6;
- the minimum number of firefighters in the hour block [10h-13h] should be 11;
- the minimum number of firefighters in the hour block [13h-16h] should be 9;
- the minimum number of firefighters in the hour block [16h-19h] should be 8;
- the minimum number of firefighters in the hour block [19h-21h] should be 3;
- the minimum number of firefighters in the hour block [20h-07h] should be 1.

Thus, the following linear programming problem was formulated:

$$\text{Minimize } F = t_1 + t_2 + t_3 + t_4 + t_5$$

$$\text{Subject to } t_1 + t_2 + t_3 + t_4 + t_5 \leq 16$$

$$t_1 + t_2 \geq 6$$

$$t_1 + t_2 + t_3 \geq 11$$

$$t_2 + t_3 + t_4 \geq 9$$

$$t_3 + t_4 \geq 8$$

$$t_4 \geq 3$$

$$t_5 \geq 1$$

$$t_1, \dots, t_5 \in \mathbb{N}_0$$

## 5. Results Analysis

Using Excel's Solver package, we used the LP Simplex resolution method, using a precision of 0.000001 as a stopping criterion. After 8 iterations, Solver has found a solution and all constraints and optimization conditions are satisfied. In this way, three types of reports were generated: the response, sensitivity, and limits.



## 5.1. Response report

Analysing Response Report 1, it shows information about the optimal solution, both for the decision variables and the cell of the objective function (Table 1). In this case, the optimal solution of the objective cell was 15, that is, only 15 firefighters are needed in total, which implies a slack in existing human resources. Besides, shift is not used in the final solution, because the software considered it more advantageous to fill shifts and to complete that hour block. However, it should be noted that in this case, weekend shifts are not being considered, nor the replacement of legally approved holidays. Usually, the firefighters assigned to the shift complete their schedule with weekend hours.

*Table 1. Solver solution*

Decisions variables	Solver solution
$t_1 = [07h-13h]$	0
$t_2 = [07h-16h]$	6
$t_3 = [10h-19h]$	5
$t_4 = [13h-21h]$	3
$t_5 = [20h-07h]$	1
<b><i>Min F</i></b>	<b>15</b>

## 5.2 Sensitivity Report

The sensitivity report provides more information about the sensitivity of the optimal solution, indicating how the constants of the problem can vary without changing the optimal solution. Table 2, “variable cells” gives the final value of each adjustable cell, its reduced cost, together with the increase or decrease of the objective function coefficient for which the current solution will remain optimal, keeping all the remaining restrictions.

*Table 2. Variable cells*

Name	Final value	Reduced cost	Objective coefficient	Permissible increase	Permissible to decrease
	0	0	1		0
	6	0	1	0	1
	5	0	1	1	0
	3	0	1		1
	1	0	1		1

Thus, the coefficient corresponding to the variable  $x_1$  shows that it is possible to increase and always maintain a margin of slack in the variable, as no firefighters are expected to work this shift in the optimal solution. The variables  $x_2$  can increase up to  $+\infty$  and decrease to 1 unit. The variable  $x_3$  may be decreased by up to one unit and not increased by any unit, while the  $x_4$  can increase up to 1 unit and not decrease anything.

Table 3. Constraints

Name	Final value	Shadow price	Right side restriction	Permissible increase	Permissible to decrease
No. of firefighters	15	0	16		1
[7-10h]	6	0	6	0	
[10-13h]	11	1	11	1	0
[13-16h]	14	0	9	5	
[16-19h]	8	0	8	0	5
[19-21h]	3	1	3	1	0
[21-07h]	1	0	1	1	1

Regarding the restrictions applied (Table 3), with special attention for the final value and the constraint:

- the total number of firefighters has not reached its maximum value (State = No Link), having already identified 1 firefighter off;
- the same happens in the time block restriction [13h-16h], in which the optimal value is above the minimum required, with a margin of 5 firefighters, this way it is possible to book additional programed work for this period, if there is need;
- in the remaining restrictions, the maximum number of firefighters has been reached, so the margin is always 0 (State = Link).

### 5.3. Limits report

Finally, the limits report is also presented (Table 4). This indicates the values that each of the decision variables can assume, not changing the remaining values and continuing to satisfy all restrictions. In this way, the lower limits are those previously defined for the decision variables, thus making it possible to maintain the use of only 15 firefighters, minimizing the total number. On the other hand, the upper limits, and taking into account that there are 16 firefighters in the fire station, the solver shows that it is possible to add 1 firefighter to the existing shifts in the problem, that is, individually and without changing the remaining shifts, each one could increase one unit.

*Table 4. Limits report*

Name	Value	Inferior limit	Objective result	Upper limit	Objective result
	0	0	15	1	16
	6	6	15	7	16
	5	5	15	6	16
	3	3	15	4	16
	1	1	15	2	16

## 6. Conclusions and future work

The scheduling of shifts is of extreme importance, since the professionals of an organization are one of the resources that have a higher cost, in this way, a correct planning can increase their productivity and satisfaction.

In this work, it was shown that the scheduling of shifts in a fire station is fundamental, contributing to a better effectiveness of the service provided and a guarantee of the correct use of human resources. Through a simple and generic approach, a brief presentation of the team of professional firefighters under analysis was carried out and a method of solving the problem and scheduling the schedules by shifts, through linear programming, was exposed. It was concluded that it would be possible to reduce 1 firefighter, with only 15 firefighters being needed, from the current 16.

However, the solution presented cannot yet be considered as a final solution, since the functions of each of the firefighters are not being considered, namely emergency ambulance service, call centre, non-urgent transport service and permanent intervention service and working hours on Saturday are not being considered.

In this way, a more complete approach to the characterization of firefighters, as well as a distinct partition of time blocks, which must include Saturdays, is essential to guarantee a complete response to the work/service developed by firefighters.

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## References

1. Sousa NM (2015) O planeamento civil de emergência no novo paradigma de segurança. Master Dissertation. Instituto Superior de Ciências Sociais e Políticas, Universidade de Lisboa.
2. Almeida A, Santos M, Lopes C, Oliveira T (2019) Bombeiros: Perceção relativa aos Fatores de Risco/ Riscos Laborais, Medidas de Proteção e Atuação dos Profissionais da Saúde Ocupacional. *Revista Portuguesa de Saúde Ocupacional* on line. 2019, volume 8, 1-17. [https://doi: 10.31252/RPSO.28.12.2019](https://doi.org/10.31252/RPSO.28.12.2019).
3. Bombeiros Voluntários Paredes, online at 27 march 2023 (<https://byparedes.pt>)
4. Costa DM (2020) Trabalho por turnos e descanso semanal a atualidade de uma antiga controvérsia. Master Dissertation, Universidade Católica Portuguesa, Porto.
5. Costa G (2003) Shift work and occupational medicine: an overview. *Occupational Medicine* Vol. 53(2): 83-88. [https://doi: 10.1093/occmed/kqg045](https://doi.org/10.1093/occmed/kqg045).
6. Henriques AS (2019) Burnout no trabalho por turnos: estudo realizado numa empresa do setor industrial têxtil. Master Dissertation, Universidade Católica Portuguesa, Viseu.
7. Tian-Shy L, Wang MJ (1991) Rotating-shift system vs. fixed-shift system. *International Journal of Industrial Ergonomics*, Vol. 7(1): 63-70. [https://doi.org/10.1016/0169-8141\(91\)90059-U](https://doi.org/10.1016/0169-8141(91)90059-U)
8. Ohayon M, Lemoine P, Briant V, Dreyfus M (2002) Prevalence and consequences of sleep disorders in a shift worker population. *J Psychosom Res* 53(1):577-583. [https://doi: 10.1016/s0022-3999\(02\)00438-5](https://doi.org/10.1016/s0022-3999(02)00438-5).
9. Kim W, Kim TH, Lee TH, Choi JW, Eun-Cheol P (2016) Park The impact of shift and night work on health related quality of life of working women: findings from the Korea Health Panel. *Health Qual Life Outcomes* 14(1):162. [https://doi: 10.1186/s12955-016-0564-x](https://doi.org/10.1186/s12955-016-0564-x).
10. McHugh M, French DD, Kwasny MM, Maechling CR, Holl JL (2020) The Impact of Shift Work and Long Work Hours on Employers' Health Care Costs. *J Occup Environ Med* 62(12):1006-1010. [https://doi: 10.1097/JOM.0000000000001994](https://doi.org/10.1097/JOM.0000000000001994).
11. Costa G (1996). The impact of shift and night work on health. *Appl Ergon* 27(1):9-16. [https://doi: 10.1016/0003-6870\(95\)00047-x](https://doi.org/10.1016/0003-6870(95)00047-x).
12. Kamila N, Barbara Ł (2021) The impact of shift work on the well-being and subjective levels of alertness and sleepiness in firefighters and rescue service workers, *Int J Occup Saf Ergon* 27(4):1056-1063. [https://doi: 10.1080/10763439.2021.1911111](https://doi.org/10.1080/10763439.2021.1911111).

10.1080/10803548.2021.1933320. Epub 2021 Jul 7.

13. Halbesleben JRB (2009) The influence of shift work on emotional exhaustion in firefighters: The role of work-family conflict and social support. *International Journal of Workplace Health Management* 2(2): 115-130. <https://doi.org/10.1108/17538350910970200>.
14. Abdalkareem, Z.A., Amir, A., Al-Betar, M.A., Ekhan, P. & Hammouri, A.I. (2021). Healthcare scheduling in optimization context: a review. *Health Technol.* 11, 445–469. <https://doi.org/10.1007/s12553-021-00547-5>
15. Hilier FS, Lieberman G (2021) *Introduction to Operations Research*. 11<sup>th</sup> Edition, McGraw-Hil.
16. Azevedo JM (2017) *Otimização da Afetação de Recursos para Prestação de Serviços*. Master Dissertation, Universidade de Aveiro. <http://hdl.handle.net/10773/24797>
17. Fontana VS, Nunes VB, Silva JM (2018) Programação linear com o uso do Excel e do Solver: uma abordagem aplicada no ensino médio. *Revista Dimensão Acadêmica* 3(2): 77-93.