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Resumo This paper attempts at inquiring the evolving nature of the relationships between architecture and mathematics. Indeed, the ancient classical link has few in common with what we experience today. But this evolution cannot be attributed only to the progress in knowledge along the ages, whether in philosophy, art, sciences, or technology. It stems from the changing and evolving commitment and meaning that both mathematicians and architects, each on their own side, have put into their work, and als...

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FROM CLASSICISM TO MODERNITY ARCHITECTURE, MATHEMATICS AND BEAUTY

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Abstract: This paper attempts at inquiring the evolving nature of the relationships between architecture and mathematics. Indeed, the ancient classical link has few in common with what we experience today. But this evolution cannot be attributed only to the progress in knowledge along the ages, whether in philosophy, art, sciences, or technology. It stems from the changing and evolving commitment and meaning that both mathematicians and architects, each on their own side, have put into their work, and also to the intellectual approach at the base of their professional researches and challenges.
In addition, the ancient relationship between mathematics and architecture was built on a common sense of a certain beauty that evolved in time, and we can wonder what kind of inheritance is left today.

Keywords: Geometry; Arithmetic; René Descartes; Gaspard Monge; Mathematical beauty; Architectural beauty.

Introduction

In Classical antiquity, together with astronomy and music, geometry and arithmetic were two of the four liberal arts (the noble disciplines that only free men could study) and the two main branches of mathematics. Both were disciplines that relied on visual representation for problem solving and didactic demonstration [Duvernoy 2018]. In Pythagorean arithmetic, numbers were shapers [1]. They were linear, planar (square, polygonal) or solid (cubic, pyramidal). We still today speak of "figurate quantities" when we speak of ancient Greek geometry. Irrational quantities, drawn as linear magnitudes, were made visible and could be precisely quantified by comparing them respect to a known length, drawn in the same scale of representation. The most famous example of irrationality and incommensurability is the question of the diagonal of the square and the irrational magnitude of $\sqrt{2}$. Plato's Meno dialog is the best explanation of the approach to mathematics through visual observation. In this dialogue, Socrates teaches mathematics to an unlearned slave, and in so doing he demonstrates the potential of graphic representation in the development of knowledge and science. The figure illustrating the graphic solution to the duplication of the square, a visual image or sensible object, allows anyone to become aware of the intelligible relationships between opposing quantities, and to figure out his or her own conclusions regarding the obvious evidence. Senses and sensorial perception are common to mankind. The capacity to observe and understand is innate and latent in anyone and does not come from a cultural privilege or a high level of education. In order to learn and progress in conscious knowledge, it is sufficient to exercise one's natural skills. The education of the neophyte or the methodology of the scientist need only concentrate on how to observe.

> *Socrates, in showing the figure, does not give the conclusion, because to see the figure, as a mathematical figure, actually means knowing how to look at it, how to read it, in short how to think it.* [Caveing, 1996]

It is notable that most of the propositions that Euclid includes in the thirteen books of the Elements are demonstrated through graphics, whose interpretation requires the visual comparison of figurate quantities. The accompanying text to each diagram guides the learner in the sensitive reading of the scheme, in the tangible evaluation of the entities, and in the appraisal of their perceptible equalities, complementarities or differences.

[1] In the sense that Lionel March applies to the expression in his essay «Architectonics of Humanism: Essay on number in architecture».

Visual perception, as a research methodology was developed independently from the ability and capacity to draw the exact shape of the mathematical objects. Greek mathematicians were unable to draw volumes in 3D. The three last books of the Elements by Euclid which deal about solid geometry and, in particular, inquire the properties of the five platonic solids, are illustrated by 2D graphics and a few very awkward 3D figures. The images illustrate in a very simplified way what the problem was and especially what was its solution. However the achievements made in classical antiquity and the progress accomplished need not corroboration by modern calculation techniques. The results coming from ancient Greece have proven to be of a high degree of precision.

One of the tricky problems of solid geometry in classical antiquity was the duplication of the cube. Many solutions were worked out by the most famous mathematicians of the time. Some of them involved the construction of new drafting tools, thanks to which the drawing of special curves was made possible. For example, the solution by Nicomedes makes use of the conchoid: a curve that he invented and whose properties he described. Nicomedes also designed the instrument that would make it possible to draw the curve. Same thing for the cissoid curve invented by Diocles, for the same purpose. The most astonishing (beautiful?) solution to the problem is the one by Archytas of Tarantum who envisioned the intersection of three solids: a torus, a right cone and a cylinder. The intersection of the three surfaces of revolution determines a point which is the sought-after mean proportional between two given lines. None of these three solids could easily be drawn on paper, let alone their intersection. So we may assume that Archytas used models while studying, and he later devised a simplified graphic image to explain his calculation to posterity. Many posthumous scholars have tried to improve the graphic image, to increase its communicative efficacy, but the computation itself did not need any improvement in accuracy, because it was exact.

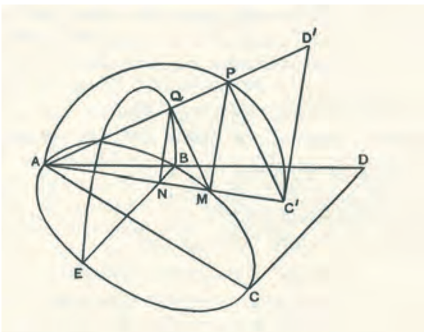

Figure 1. Solution to the duplication of the cube by Archytas of Tarantum. [Heath 1981].
Note that the three solids: torus, cone and cylinder do not show.

Beauty in ancient mathematics

In ancient mathematics, some special numbers and geometric shapes are considered to be beautiful. The concept of beauty is connected to the concepts of order and symmetry.

> *…they are in error who assert that the mathematical sciences tell us nothing about beauty or goodness; for they describe and manifest these qualities in the highest degree, since it does not follow, because they manifest the effects and principles of beauty and goodness without naming them, that they do not treat of these qualities. The main species of beauty are orderly arrangement, proportion, and definiteness; and these are especially manifested by the mathematical sciences* (Aristotle - Metaphysica, 13-1078a.)

Order is obtained when shapes are regular. Equilateral, equiangular shapes are regular shapes, and therefore beautiful. In modern terms we would say "simple". The square is the paradigm of order in planar geometry. The squaring of an irregular shape is a recurrent procedure in Greek geometry, and Euclid shows in the Elements how to transform any triangle and any quadrangle in perfect squares without altering the dimension of their areas. The squaring of many figures makes it possible to compare them, and to appreciate visually (in addition to calculate) their different sizes and numerical properties.

Symmetry is not only a graphic concept linked to reflection in respect to an axis, but it is an arithmetical concept of commensurability and/ or similarity of proportions and proportional ratios. The beautiful shapes are those whose various dimensions, expressed in natural integers, are all commensurable between them, because they share a common divisor, i.e. a common unit.

Mathematical utmost beauty is achieved when arithmetic and geometry combine to describe special shapes. The paradigm of order and symmetry is the so-called "Pythagorean" triangle. The Pythagorean triangle is defined both by its geometrical shape which is a rectangle triangle, and by the dimensions of its sides which are natural integers and the squares built on its sides are also commensurable between them because the sum of the two lesser ones gives the square on the hypotenuse. We thus have a double commensurability of the sides and of the squares on the sides. Only some triplets of natural integers can produce this double commensurability and triangles drawn and built with these numbers assigned to their sides are all going to be rectangle triangles.

Plato speaks of mathematical beauty in absolute terms. Both geometrical shapes and proportional systems may be beautiful. In the Timaeus he states that the rectangle triangle which comes from halving the equilateral triangle is the most beautiful of all triangles, the regular polyhedra are the fairest of all bodies, and the geometrical proportion is the most beautiful of all.

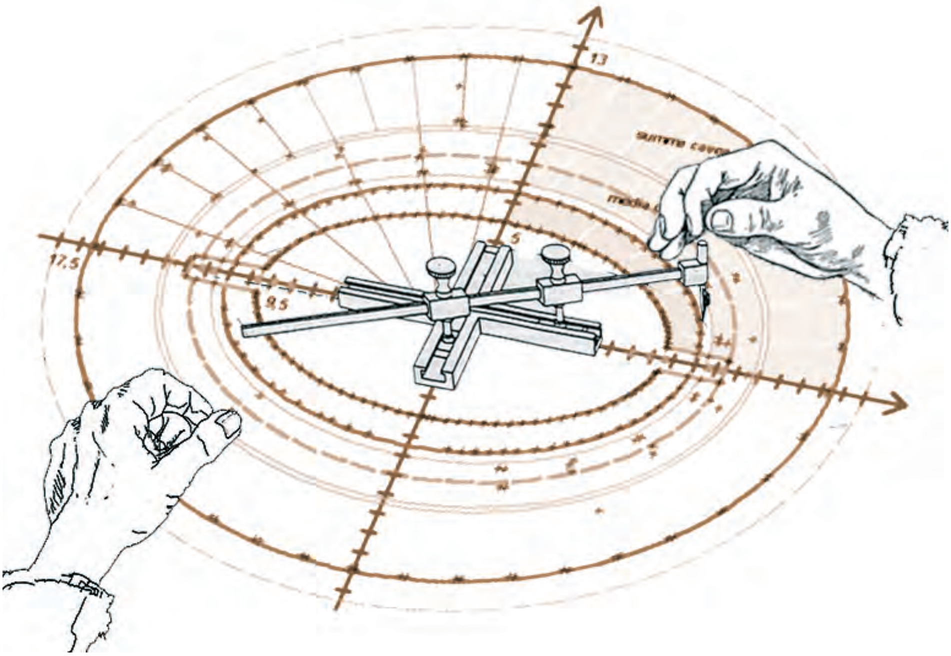

Figure 2. Conic curves and Roman architecture: the ellipses of Pompeii's amphitheatre geometrical diagram. Unknown designer. Built in 80 B.C. [Duvernoy 2006] (drawing by author)]

Architecture and mathematics in Italian Renaissance

The concept of beauty in mathematics has been transmitted from classical antiquity to Renaissance architects mainly through Vitruvius treatise *De Architectura Libri Decem*. Built monuments and architecture treatises that have been produced from the Fifteenth century on, show how this concept was applied to design.

Referring to Pythagoras's musical theory Leon Battista Alberti (1404 –1472) states that the same numbers that are able to produce delightful music can also produce beautiful architecture and pleasure to the eyes [Alberti 1988]. Also, by stating that the favourite shape of nature is the circle, he suggested that temples should be designed on polygonal diagrams and therefore set off the typology of centrally planned churches.

Renaissance humanists also included in the many mathematically beautiful items the peculiar proportion that Euclid used to name the "extreme and mean ratio". Euclid gives the definition of "extreme and mean ratio" at the beginning of Book 6 of the Elements (definition number 3) and he gives the rule of how to divide a line in extreme and mean ratio in Book 6, proposition XXX. In Euclid's day, this proportion was only a necessary calculation tool to define the ratios between the various sides of the five regular polyhedra inscribed in a sphere, which are linked by different kinds of incommensurable relationships.

Around 1509, the mathematician Luca Pacioli (1445–1517) wrote a full treatise dedicated to the "extreme and mean ratio" entitled *De Divina Proportione*. In this book he explained at length the thirteen properties of this special proportion, each being highlighted by an epithet such as "bizarre", "wonderful", "supreme", "superb", "incomprehensible", "magnificent", etc… No word was enough for him to describe this marvel of mathematics. Today the "divine proportion" is commonly named "the golden section" and its arithmetical value is the "golden number", which is an irrational magnitude [Pacioli 1982]. It is surprising to note that no architectural treatise of the Renaissance lists the golden section among the beautiful proportions. However, there is evidence of its use in architectural design, showing that interactions between mathematics and architecture did exist beyond what is reported in the contemporary architectural literature. In fact, more than the literary sources, the extent and value of the relationship between architecture and mathematics in the Fifteenth and Sixteenth centuries have to be found in the analysis of built monuments. During the Renaissance many architectural treatises were written by the most prominent architects of the time, however the mathematical notions that they include in their texts are not many, and they do not inform us on how learned in mathematics the authors were. Most of the treatises start with some pages showing drawing tricks, standard calculation rules inspired from Euclidean geometry, and the list of the beautiful musical proportions. Those pages appear more as a selection of some "tricks of the trade" rather than a true theoretical scientific knowledge. Sebastiano Serlio himself (1475–1554 ca.) made a mistake while reporting how to draw an accurate perspective construction [Xavier 1997]. Even Leonardo da Vinci (1452 –1519), after having studied Euclid's Elements under the guidance of Luca Pacioli claimed to have solved the problem of the squaring of the circle while instead, he did not. Many mathematical notions are included *in the book by Alberti De Ludi Mathematicarum*, a short booklet written at the request of the Duke Meliaduse d'Este, but those are, once again, only known tricks to calculate various quantities: distances, lengths, depths, speeds, etc… [Williams 2010]. The scientific theory between the computations is not addressed.

In the introduction to his book entitled *Lives of the Most Excellent Painters, Sculptors, and Architects*, Giorgio Vasari (1511-1574) explains that architecture is one of the "three arts of drawing" together with painting and sculpture [Vasari 1991]. The reading of the book indeed shows that most architects were also talented painters or sculptors. Just like Vasari himself, many of them started their careers as painters before switching to architectural design: Donato Bramante (1444 –1514), Michelangelo Buonarroti (1475 –1564), Giulio Romano (1499 – 1546), etc…

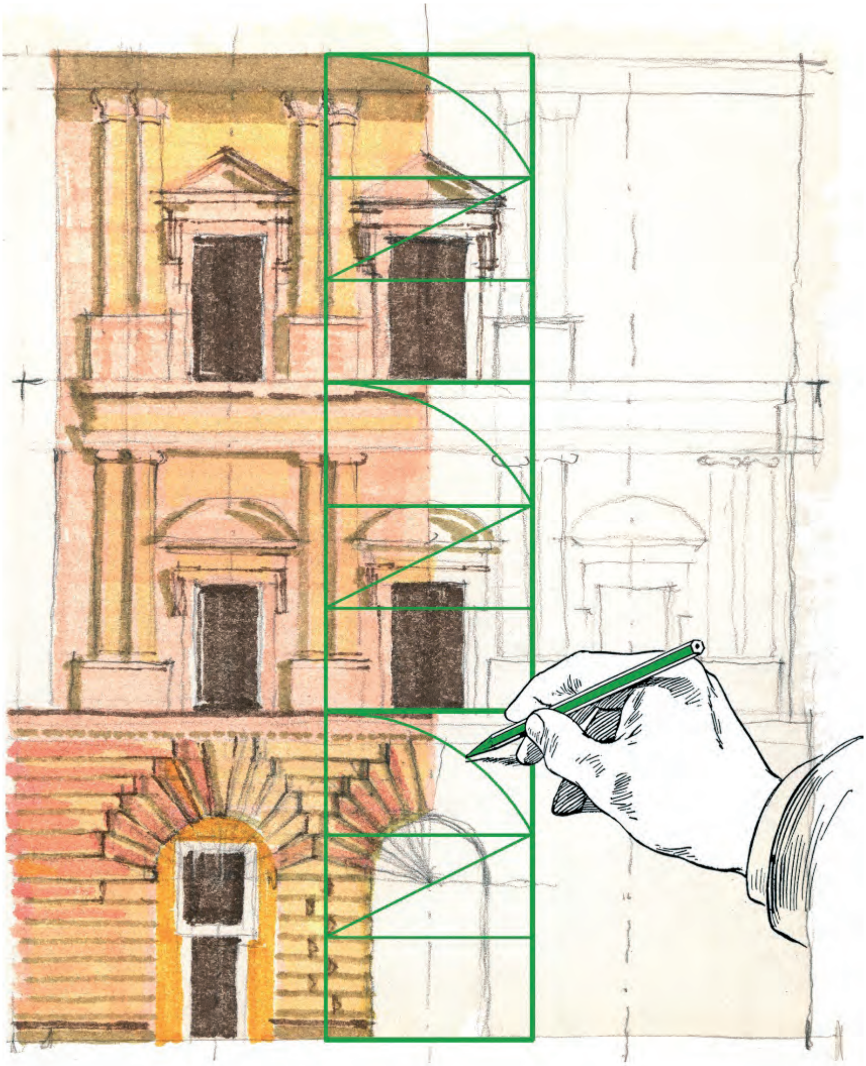

Figure 3. The Divine Proportion and Palazzo Uguccioni in Florence-Italy. Unknown designer. Built in the mid Sixteenth century. (drawing by author)

Piero della Francesca (1415-1492) was a Renaissance polymath. Equally trained in both disciplines, he is remembered both as painter and as a mathematician. Besides two books in mathematics (*Trattato d'Abaco and Libellus de Quinque Corporibus Regularibus*) he wrote the seminal book: *De Prospectiva Pingendi*, published in 1482 which is the very first scientific treatise where art and mathematical science meet to produce the theorization of perspective drawing.

Architecture and mathematics in the Seventeenth century

A new approach to the relationship between architecture and mathematics appears in the mid XVIIth century with Guarino Guarini (1624-1683), the first scholar to write both a full treatise on mathematics and a treatise on architecture.

His treatise entitled *Euclides Adauctus et Methodicus Mathematicaque Universalis* (published in Turin in 1671) of more than 700 pages written in Latin, is not only a commentary of Euclid's Elements but a compendium of all the knowledge coming down from the Greek and Roman mathematicians, and the later researches by scientists of the following centuries, up to his day. In his book Guarini quotes and refers to a number of recent publications on specific mathematical topics. This treatise alone testifies to the extent of his knowledge in theoretical mathematics which goes much beyond the knowledge of any other Italian architect of the Renaissance and Baroque period. Two of his other writings deal with architecture, both written in Italian. The first is *Modo di Misurare le Fabriche*, published in 1674. The second, *Architettura Civile*, was left unfinished and published posthumously fifty years after his death, by Bernardo Vittone (1704 –1770).

However, while Guarini is still studying Euclid, the French philosopher René Descartes (1596-1650) is opening the way to a modern mathematical science. Descartes marks a turning point in mathematical history. He would claim himself a philosopher, but his worldwide fame comes from mathematics. In 1637 he publishes in Leiden (Holland) the seminal treatise: *Discourse on the Method of Rightly Conducting One's Reason and of Seeking Truth in the Sciences*, written in French. The "discourse" is completed by three appendixes entitled *Dioptrics, Meteores, and Geometry*, which are three case studies of the method for seeking truth in science. The one with which we are concerned is *Geometry*. It had to have a fundamental impact on the science of geometry. Today we all speak of "Cartesian space" "Cartesian axes" and "Cartesian planes" but this impact was far from being immediate. The new method for problem solving invented by Descartes was not fully understood by contemporary mathematicians and was fiercely criticized. In the booklet *Geometry*, Descartes takes Pappus problem as his first case study. This problem, also named "the four lines problem", originated in the time in which Greek geometers were looking for a general solution to the trisection of any angle. The goal is to find point C so that the product of the distances from C to two of the given lines is equal to the product of the distances from C to the two other lines. The lines extending from C on which the distances are taken must cross the given four lines at constant angle. When only three lines are concerned, the product of the distances from C to two of the lines must be equal to the square of the distance from C to the third line.

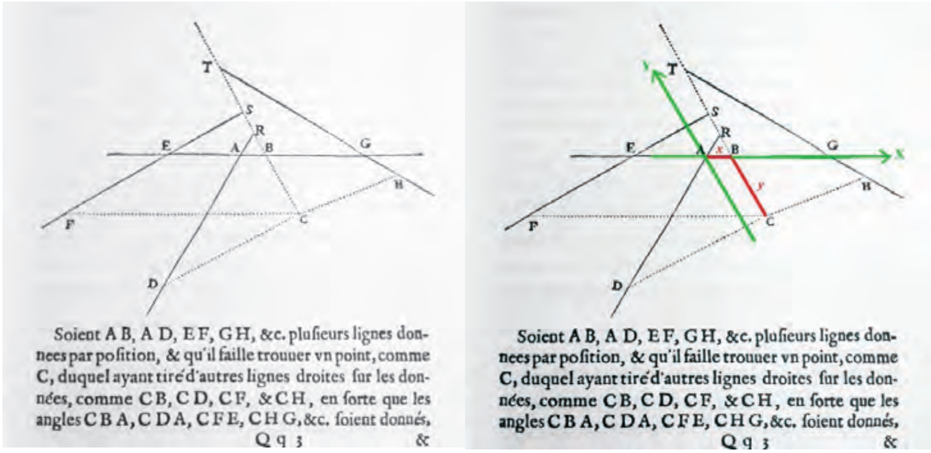

Figure 4. From *Descartes, La Géométrie*. Solution to Pappus problem. Original drawing and figure with superimposition of the "cartesian axes". [Descartes 1996].

Descartes' figure only shows the problem but does not give the solution. The solution is not drawn on the figure. So the modern reader in search for the very first drawing in which cartesian axes would actually be represented is going to be disappointed. No such thing happens here. The solution to the geometric problem will come from the carefully manipulated set of algebraic equations that are handed to the reader. Probably this explains why Descartes was severely rebuked by some colleagues such as Etienne Pascal (1588-1651), Pierre de Fermat (?-1665) and Roberval (1602-1675). The strength of the method and its capacity of problem solving was not understood immediately by all, and some historians of mathematics even suggest that it was not fully envisioned by Descartes himself. What is worth noticing however, is the switch from visual and graphic geometry to analytic geometry. Of course, algebra and analysis had started to be developed much before Descartes, but he was the one who made them universal. Descartes is not a "visual" person and asserts it by written. He is not afraid to say that the ancient Greek problem solving method through graphics is "tiring".

> *The analysis of the ancients … is always so bound to observation of figures that it cannot exercise understanding without exhausting imagination* [Descartes 1996] [2]

[2] In the original text: *"L'analyse des anciens… est toujours si astreinte à la considération des figures qu'elle ne peut exercer l'entendement sans fatiguer beaucoup l'imagination."*

With Descartes, mathematics becomes abstract. Being published in French the "*Discours de la Méthode*" could not be read by an international readership, so it was republished in Latin in Amsterdam in 1644 but this edition did not include the text on geometry. The first Latin translation of "*La Géométrie*" was released in Leiden in 1649, only one year before his author's death.

Who was the first to figure out that this method was based on the postulation of two coordinate axes (X and Y) having origin at point O… we don't really know. Surely enough, the full theorization of Descartes' geometric method came after his death. The modern notion of coordinate system as a starting point for geometric problem solving appears by written for the first time in the book *Introductio in Analysin Infinitorum* by Leonhard Euler (1707-1783), published in 1748. Leonhard Euler is also credited for having extended the method from 2D to 3D, adding the Z axis. Descartes cannot be credited for having invented algebra and analysis, that were inquired long before him, nevertheless mathematics changed from then on. Visual geometry was "discarded" in favour of analysis and abstraction. Sensorial perception was no longer the main tool for mathematical research.

Modern times

What sense of beauty such an approach to mathematical research will generate? And can this sense of beauty be shared with other scholars or researchers? In the seventeenth century architects and designers like Guarino Guarini were still referring to Euclidean geometry, the one that had always been a support of architectural design so far; both being visual disciplines. But gradually, modern mathematics would replace traditional geometry. Modern mathematicians still have a certain sense for beauty and elegance. Certain equations are considered "beautiful", or some demonstrations are more "elegant" than others. Can those feelings continue to reflect in architectural design?

In addition, the switch to abstraction and analysis unfortunately created a sort of hierarchical relation between mathematics and architecture, where architecture became an application field of mathematics. This new kind of relationship had already started to be claimed by some mathematicians. The French Jesuit Claude-François Milliet Dechales (1621–1678) wrote a textbook entitled *Cursus seu Mundus Mathematicus* which became quite influential. His textbook is a sort of encyclopaedia in four volumes in which the architecture topic is covered in 23 pages and reduced mainly to a list of the classical orders. Architecture and construction together cover 140 pages if we include the chapters on timber structures and stone cutting. This approach to architectural design indeed introduces a new approach to the relationship between architecture and mathematics. It suggests that being learned in mathematics is a sufficient education to be able to design.

This approach is indeed going to develop rapidly, especially outside Italy. The change is noticeable in Northern Europe where the men who became the most famous architects were actually trained in mathematical sciences, starting their careers as engineers, astronomers, geometers, etc… As such, they were considered to be able to solve any kind of problem including those related to architectural design. They were therefore appointed by the kings to design and build monuments. This was the case in France for François Blondel (1618–1686), or Claude Perrault (1613–1688). The former, who was an engineer and a diplomat, became a professor of mathematics, and a professor of architecture, at the French "Académie Royale d'Architecture" that he directed for some years. The latter was first and foremost a physician and anatomist, and was nevertheless asked to write a French translation of Vitruvius treatise. In the same years in England, the king asked Christopher Wren (1632–1723) - an acclaimed mathematician – to take care of the reconstruction of London after the Great Fire of 1666. His colleague Robert Hooke (1635–1703) was part of the task force.

It was not before the turn to the Nineteenth century that a branch of mathematics returned to visual procedures, thanks to Gaspard Monge (1746–1818). Monge, born in France in the mid Eighteenth century, was a brilliant mathematician equally skilled in algebra, analytic geometry, and differential geometry. He had both an abstract and a visual intelligence. Unlike Descartes he could think in "numbers" as well as in "images". Indeed he is credited for having theorized descriptive geometry: the science of graphic representation of 3D objects and 3D space through a system of multi-views. With some fellow French revolutionaries, he co-founded the famous Parisian engineering school, "Ecole Polytechnique", in 1794. In the preface of the textbook that he wrote for his students, entitled Géométrie Descriptive, first published in 1799, he says:

> *… any analytic demonstration can be seen as the script of a show in descriptive geometry.* [Monge, 1989] [3]

The word "show" ("spectacle" in the French text) is particularly pleasing because it brings back the sense of awe and beauty that an image - more than a set of equations - can inspire to the scholar.

[3] In the original text: … *chaque opération analytique peut être regardée comme l'écriture d'un spectacle en géométrie.*

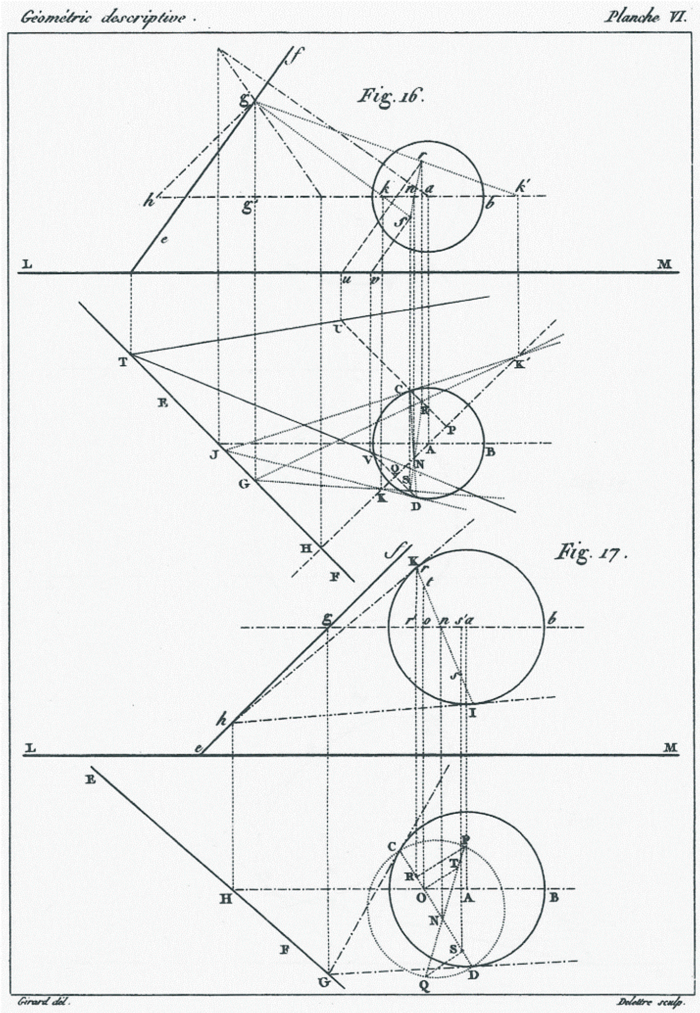

Figure 5. Two solutions to the same problem: draw a plane tangent to a given sphere passing through a given line. Monge says that the second solution is "much more elegant", because more concise. [Monge 1989]

Conclusion

In the very abstract world of contemporary mathematics, the concept of mathematical beauty is not stated by scientists as clearly as it was in classical culture, and is therefore less easily shared. Today, when we, architects, use digital technology, computer aided drafting, visual programming language and other contemporary mathematical design tools, how do we address the concept of beauty? Is there a mathematical beauty in the software that we use that we are aware of? Is there an architectural beauty in the products that software made possible?

Is there still a common sense of beauty shared by mathematicians and designers or artists?

Bibliographic references:

Alberti, L.B. (1988). *On the art of building in ten books* (1452). J. Rykwert, N. Leach, R. Tavernor (eds). Cambridge MA, MIT Press.
Caveing, Maurice-Louis, Evelyne Barbin (1996). *Les philosophes et les mathématiques*. Paris, Ellipses.
Descartes, René (1996). *La Géométrie*. Paris, Presses universitaires de France.
Duvernoy, Sylvie (2018). "Classical Greek and Roman Architecture: Mathematical Theories and Concepts". In: Sriraman B. (eds) *Handbook of the Mathematics of the Arts and Sciences*. Springer, Cham.
Duvernoy, Sylvie, Paul L. Rosin (2006). "The compass, the ruler and the computer" in *Nexus VI - Architecture and Mathematics*, eds. S. Duvernoy, O. Pedemonte. Torino, Kim Williams Books.
Heath, Sir Thomas (1981). *A History of Greek Mathematics*. New York, Dover.
March, Lionel (1998). *Architectonics of Humanism: Essay on number in Architecture*. London, Academy Editions.
Monge, Gaspard (1989). *Géométrie descriptive*. Sceaux, Jacques Gabay.
Pacioli, Luca (1982). *De divina proportione.* Milano, Silvana.
Vasari, Giorgio (1991). *Lives of the Most Excellent Painters, Sculptors, and Architects*. Translated J.C. and Peter Bondanella. Oxford, Oxford University Press.
Williams, Kim, Lionel March, Stephen R. Wassell (2010). *The Mathematical Works of Leon Battista Alberti*. Basel, Birkhäuser.
Xavier, João Pedro: *Perspectiva, Perspectiva Acelerada e Contraperspectiva*. Porto, FAUP. 1997.

Web reference:

Aristotle, 350 B.C.: Metaphysics: http://classics.mit.edu/Aristotle/metaphysics.html